Significant Digits (also known as significant figures)

Rules for deciding which digits are significant:

1. Nonzero digits are always significant.
2. All final zeros after the decimal point are significant.
3. Zeros between two other significant digits are always significant.
4. Zeros used solely for spacing the decimal point are ***not*** significant.

Examples:
Using the rules above, 0.0340 mm has 3, 960 kg has 2, and 70,080 s has 4 significant figures.

123.479 m + 35.8 m + 5.32 m = 164.599 m = 164.6 m
(In addition or subtraction, work with the numbers as they are written, but the final answer must have the same level of precision as the least precise number – in this case, precision to the tenths place.)

$$\frac{123.479\ \text{kg}}{35.8\ \text{m}^3} = 3.449134078\ \frac{\text{kg}}{\text{m}^3} = \underline{3.45}\ \frac{\text{kg}}{\text{m}^3}$$

(In multiplication or division, work with the numbers as they are written, but the final answer must have the same number of significant digits as the number with the least amount of digits – in this case, 3 significant digits.)

$$\frac{7.48\ \text{m}^3}{3.6\ \text{m}} = \underline{2.1\ \text{m}^2}$$

(In multiplication or division, units are multiplied and divided like numbers.)

Exercises

1. **State the number of significant digits in each of the following measurements.**

   a. 32.06 kg ________ b. 0.02 km ________ c. 5400 m ________ d. 2006 s ________

   e. 2.9910 m ________ f. 5600 km ________ g. 0.00670 kg ________ h. 809 g ________

2. **Solve the following problems and give the answers to the correct number of significant digits. (All numbers must be written with the correct units.)**

   a. 324.54 cm - 25.6 cm = ________________

   b. 28.9 g + 300.25 g + 2.945 g = ________________

   c. 82.3 m x 1.254 m = ____________________

   d. $(1.2 \times 10^6\ \text{m})(3.25 \times 10^4\ \text{m})$ = __________________

   e. $\frac{32.6\ \text{kg}}{125.4\ \text{L}}$ = ___________________________

   f. $\frac{4.24 \times 10^{-3}\ \text{kg}}{2.2 \times 10^{-4}\ \text{L}}$ = ____________________________

   g. $\frac{2.647\ \text{m}}{2.0\ \text{m}}$ = ___________________________

   h. 5.25 cm x 1.3 cm = __________________________

   i. 9.0 cm + 7.66 cm + 5.44 cm = ___________

   j. 10.07 g - 3.1 g = _____________________________

Scientific Notation

Scientific notation is a compact way of writing large or small numbers while using only significant digits and powers of ten.

Examples:

0.00265 written in scientific notation would be $\underline{2.65 \times 10^{-3}}$.
(The negative three power of ten indicates that the decimal point should be moved three places to the left.)

7.68×10^{5} expanded would be <u>768,000</u>.
(The positive power of five indicates that the decimal point should be moved five places to the right. In this case, zeros are needed as placeholders.)

$$\frac{9.6 \times 10^{7}}{3.2 \times 10^{4}} = \underline{3.0 \times 10^{3}}.$$

(Divide the decimals while keeping the correct number of significant figures. When dividing powers of ten, subtract the bottom power of ten's exponent from the top power of ten's exponent. If multiplying, add powers of ten.)

$(4.5 \times 10^{-2}) + (8.2 \times 10^{-3}) = 4.5 \times 10^{-2} + 0.82 \times 10^{-2} = 5.32 \times 10^{-2} = \underline{5.3 \times 10^{-2}}$.
(Before adding or subtracting numbers in scientific notation, all numbers need to have the same power of ten)

Exercises

Write the following numbers in scientific notation.

1. 156.90 ___________________________ **2. 0.0345 ___________________________**

3. 0.00890 __________________________ **4. 560 ______________________________**

5. 43,200 ___________________________ **6. 4,320,000 ________________________**

7. 0.00065 __________________________ **8. 101.35 ____________________________**

Expand the following numbers.

9. 1.54×10^{4} _____________ **10. 2.5×10^{-3} _____________** **11. 5.67×10^{-1} _____________**

Solve the following and put your answer in scientific notation. (With the correct number of significant figures.)

12. $\frac{6.6\text{x}10^{-8}}{3.3\text{x}10^{-4}} =$ ____________ **13.** $\frac{7.4\text{x}10^{10}}{3.7\text{x}10^{3}} =$ ____________ **14.** $\frac{2.5\text{x}10^{8}}{7.5\text{x}10^{2}} =$ ____________

15. $(2.67 \times 10^{-3}) - (9.5 \times 10^{-4}) =$ ____

16. $(1.56 \times 10^{-7}) + (2.43 \times 10^{-8}) =$ ____

17. $(2.5 \times 10^{-6})(3.0 \times 10^{-7}) =$ ____

18. $(1.2 \times 10^{-9})(1.2 \times 10^{7}) =$ ____

19. $(2.3 \times 10^{4})(2.0 \times 10^{-3}) =$ ____

Factor-Label Unit Conversion

Common Prefixes:

nano has the symbol, n, and means 10^{-9} (or 0.000000001)
micro has the symbol, μ, and means 10^{-6} (or 0.000001)
milli has the symbol, m, and means 10^{-3} (or 0.001)
centi has the symbol, c, and means 10^{-2} (or 0.01)
kilo has the symbol, k, and means 10^{3} (or 1000)

Making Unit Conversions

250 μm = ______________ m

$$(250\ \mu m)\left(\frac{10^{-6}\ m}{1\ \mu m}\right) = 2.5 \times 10^{-4}\ m$$

or $$(250\ \mu m)\left(\frac{1\ m}{10^{6}\ \mu m}\right) = 2.5 \times 10^{-4}\ m$$

FACTOR-LABEL UNIT CONVERSION

An easy way to change from one unit to another is by using conversion factors. To convert a speed given in km/h to m/s, you must first change kilometers to meters, then hours to seconds. The value of a quantity does not change when it is multiplied by 1. Any quantity divided by its equivalent equals one. Since 1000 m = 1 km and 3600 s = 1 h, we can make the following conversion factors.

$$\left(\frac{1000\ m}{1\ km}\right) = 1 \quad \text{and} \quad \left(\frac{1\ h}{3600\ s}\right) = 1$$

Therefore, to change a speed in km/h to m/s, first multiply it by an appropriate distance conversion factor and then by a time conversion factor. For example, 120 km/h becomes

$$\left(\frac{120\ km}{1\ h}\right)\left(\frac{1000\ m}{1\ km}\right) = 120{,}000\ \frac{m}{h}$$

Then, $$\left(120{,}000\ \frac{m}{h}\right)\left(\frac{1\ h}{3600\ s}\right) = 33\ \frac{m}{s}$$ (The converted number should have the same number of significant digits as the original number.)

For the problems below, show <u>all</u> conversion factors. Show all work. Do <u>*not*</u> use the conversion factor feature on your calculator to do these problems.

1. Carry out the following conversions using the prefix information shown above.

a. 35 nm (———) = ______________ m

b. 450 cm (———) = ______________ mm

c. 1500 μg (———) = ______________ g

d. 250 km (———) (———) = ______________ cm

e. 346 ms (———) = ______________ s

f. 543 mg (———) (———) = ______________ kg

g. 4008 g (———) = ______________ mg

h. 239 mm (———) (———) = ______________ cm

i. 48 mL (———) = ______________ L

j. 38 kg (———) (———) = ______________ mg

Unit Analysis

Unit analysis involves the placement of units in an equation to check for unit agreement on both sides of the equals sign. When substituting values into an equation in physics, **you <u>must</u> state the units as well as the numerical values**. Including units in your calculations helps you keep units consistent throughout and assures you that your answer will be correct in terms of units. The proper units for variables are included in the table below.

Quantities	Units
(d) displacement	**m (meters)**
(v_o) original velocity **(v_f) final velocity** **($\bar{v}$) average velocity**	$\frac{m}{s}$ **(meters per second)**
(a) acceleration	$\frac{m}{s^2}$ **(meters per second squared)**
(t) time	**s (seconds)**

Examples

Equation with Unit <u>Agreement</u>

The equation being inspected is: $\bar{v} = \frac{d}{t}$ **, its units indicate that** $\frac{m}{s} = \frac{m}{s}$

The units on the left <u>are equal</u> to the units on the right of the equals sign. This average velocity equation is ***<u>correct</u>*** in terms of its units.

Equation with Unit <u>Disagreement</u>

The equation being inspected is: $v_f = v_o^2 + 2ad^2$ **, its units indicate that** $\frac{m}{s} = \left(\frac{m}{s}\right)^2 + \left(\frac{m}{s^2}\right)(m)^2$

(<u>Notice that **coefficients,** such as the number **2,** in the equation **are ignored** in dimensional analysis.</u>)

The units on the left are $\frac{m}{s}$ **,** but the units on the right turn out to be $\frac{m^2}{s^2} + \frac{m^3}{s^2}$

The units do **not** agree. This final velocity equation is ***<u>incorrect</u>*** in terms of its units.

Problems

Use the method described above to determine if the following equations have unit agreement.
Show your work and write "*correct*" next to the equation if it is correct and "*incorrect*" if it is incorrect in terms of its units.

1. $v_f = v_o + at$

2. $\bar{v} = \frac{1}{2}(v_o + v_f)$

3. $d = v_o t + \frac{1}{2}at$

4. $v_f^2 = v_o^2 + 2ad$

5. $d = \sqrt{v_o t^2 + \frac{1}{2}at}$

Solving for Variables

PROBLEM SOLVING

When solving motion problems, or, indeed, any physics problem, use an orderly procedure like the one listed below.

1. **Identify the quantities that are given in the problem.**
2. **Identify the quantity that is unknown, the one you have to find.**
3. **Select the equation that contains the given and unknown quantities.**
4. **Solve the equation for the unknown quantity using algebra.**
5. **Substitute the values given in the problem, along with their proper units, into the equation and solve it.**
6. **Check to see if the numerical value of your answer is reasonable *and* make sure that the answer has the correct units.**

The table below shows four common motion equations. Refer to the previous page of the packet to find the meaning of each variable.

Equations of Motion for Constant (or Uniform) Acceleration	
Equation	**Quantities Related**
$v_f = v_o + at$	v_o, v_f, a, t
$d = ½(v_f + v_o)t$	v_o, d, v_f, t
$d = v_o t + ½at^2$	v_o, d, a, t
$v_f^2 = v_o^2 + 2ad$	v_o, d, v_f, a

Example

Solve for each variable in the equation below. **The variable should be placed alone on the left side of the equation.**

$v_f = v_o + at$ (It is already solved for v_f, so it only needs to be solved for v_o, a, and t.)

$v_o = v_f - at$, $a = \frac{v_f - v_o}{t}$, $t = \frac{v_f - v_o}{a}$

Problems

1. In the equation, $d = ½(v_f + v_o)t$, solve for v_f and **t**.

2. In the equation, $d = v_o t + ½at^2$, solve for v_o and **a**.

3. In the equation, $v_f^2 = v_o^2 + 2ad$, solve for v_f, v_o, and **d**.

Solving for Variables

Examples

Solve the following equation for c.

$\mathbf{E = mc^2}$

(Divide both sides by m**)**

$\mathbf{\frac{E}{m} = c^2}$

(Take the square root of both sides and there will be a positive *and* a negative solution for c)

$\pm\sqrt{\frac{E}{m}}$ **= c** ***or,*** **finally, c =** $\pm\sqrt{\frac{E}{m}}$

Solve the following equation for k.

$\mathbf{\frac{3\sqrt{k}}{g} = d}$

(Multiply both sides by g)

$\mathbf{3\sqrt{k} = gd}$

(Divide both sides by 3)

$\sqrt{\mathbf{k}} = \frac{\mathbf{gd}}{\mathbf{3}}$

(Square both sides of the equation)

$\mathbf{k = \frac{g^2d^2}{9}}$

Problems

Solve the following equations for the variable(s) requested.

1. $E = ½mv^2$ for **m** <u>and</u> **v**

2. $P = \frac{Fd}{t}$ for **d**

3. E = mgh for **h**

4. $E = hf - W_o$ for $\mathbf{W_o}$ <u>and</u> **h**

5. $\frac{s_o}{s_i} = \frac{d_o}{d_i}$ for $\mathbf{s_i}$

6. $ax^2 + b = c$ for **x**

Solve for *x* in the following problems.

7. $\frac{3x}{y} = \frac{6g}{b}$ ________________________

8. $\frac{2x^2}{3} = dg$ ________________________

9. $d = \frac{t}{x}$ ________________________

10. $\frac{2\sqrt{x}}{c} = y$ ________________________

Graphing

Plotting Graphs

These steps will help you plot graphs from data tables.

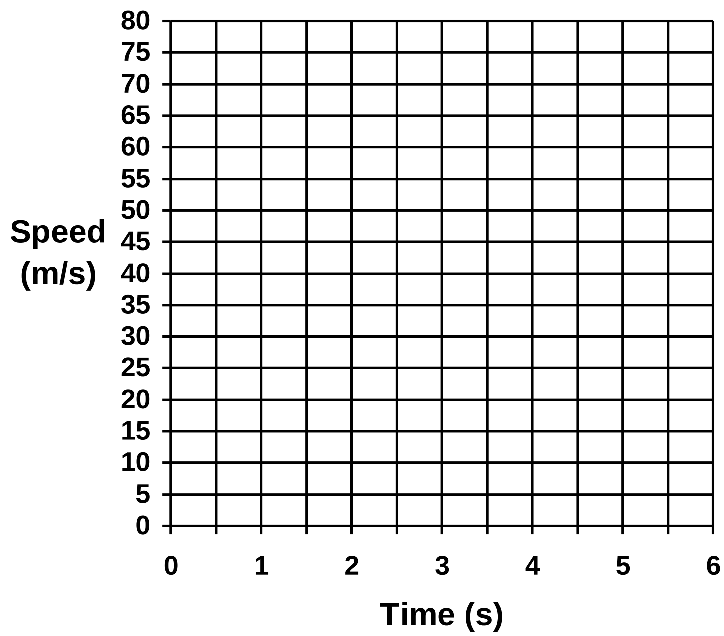
1. **Your graph should be titled Y vs. X (or the vertical information versus the horizontal information).**
2. **Decide on the scales needed for the x and y axes. Choose scales that will spread out the data. Do <u>not</u> choose scales that compress the data points into a tiny portion of the graph paper. *Your graph should fill up the graph paper.***
3. **Number and label the x and y axes (including necessary units).**
4. **Draw the best straight line (using a straight edge) or smooth curve that <u>passes through as many data points as possible</u>. Do <u>not</u> just connect the data points together with a series of straight line segments.**

Graphing Data Values

The steps listed above were followed to set up the plotting of the data shown below.

Time (s)	Speed (m/s)
0	4
1	15
2	20
3	37
4	55
5	59

Speed versus Time



Exercises

1. **Plot the data values on the graph provided above and draw one *straight* line that best fits the data (use a <u>straight edge</u> to make the line).**

2. **What is the slope of the line of best fit (find the *number*)? ______________________ m/s^2 *(Slope = rise/run)***

3. **What is the speed at 3 s? ____ m/s**

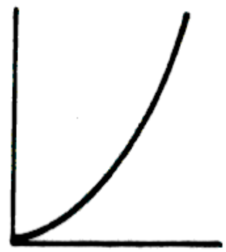
4. **Using the graph, what is the speed at 6 s? ____________________________________ m/s**

5. **At what time is the speed 20 m/s (*using your graph*) ____________________________ s**

Graphing

There are three relationships that occur frequently in physical processes. They are depicted in the three graphs shown below.

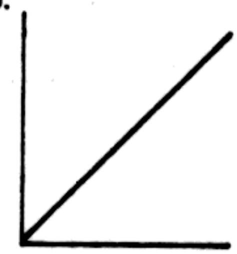
a.



a. This graph indicates that y varies directly with the square of x. The graph is a parabola (with the negative x portion not shown) and follows the equation $y = kx$.

(k represents a constant)

b.



b. This graph indicates that y varies directly with x. The graph is a straight line and follows an equation of the form $y = kx$.

(In math, this is usually written as $y = mx + b$)

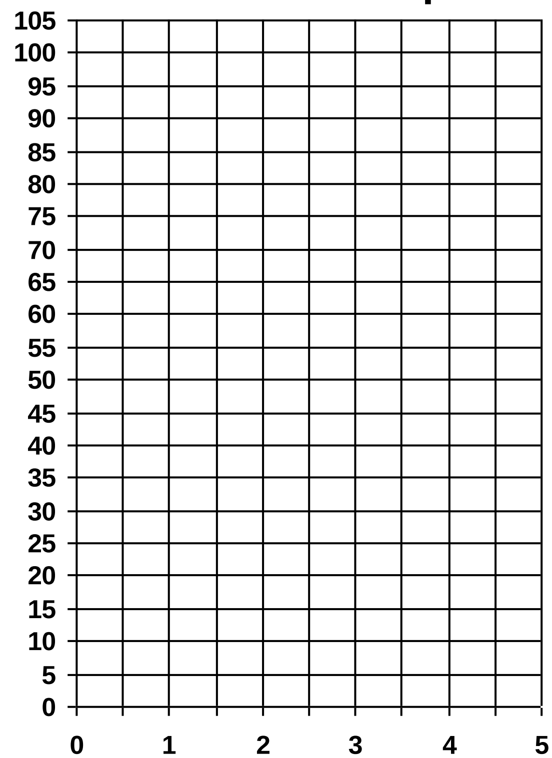
c.

c. This graph indicates that y varies inversely with x. The graph is a hyperbola and follows an equation of the form $y = k/x$.

Speed (m/s)	Radius (m)
0	0
1	4
2	16
3	36
4	64

Radius versus Speed

Radius (m)



Speed (m/s)

Exercises

1. **Plot the data values on the graph provided above and connect the points together with a smooth curve that follows the data.**

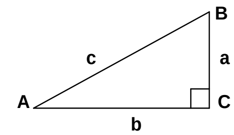
2. **Approximately what is the radius when the speed is 5 m/s? ________________________m.**

3. **This type of curve is known as a ____.**

4. **This graph follows an equation of the form radius = k(speed)^2. Radius and speed squared are __________________________________ related.**

Trigonometry

Trigonometry is an extremely useful branch of mathematics that deals with the relationships between the sides and angles of right triangles.



The Pythagorean Theorem **is a method for finding missing sides of right triangles. It states that the hypotenuse (the longest side) is related to the other two sides of a triangle by the following equation:**

$$c^2 = a^2 + b^2$$

(where c is the length of the hypotenuse and a and b are the lengths of the other two sides)

The three commonly used *trigonometric functions* are

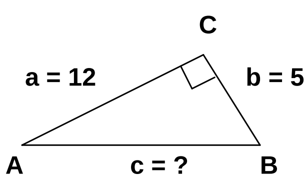
The sine: $\sin \angle A = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{a}{c}$

The cosine: $\cos \angle A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{b}{c}$

The tangent: $\tan \angle A = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{a}{b}$

(Memorizing **SOH CAH TOA** is one way of memorizing these trigonometric functions.)

Examples – answers with 3 sig. figures.

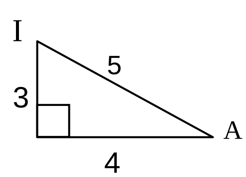


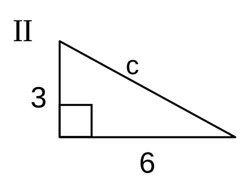
By the Pythagorean Theorem, c = 13

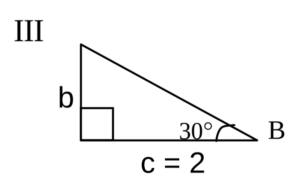
$\cos \angle A = \dfrac{12}{13} = 0.923$ $A = \cos^{-1}\left(\dfrac{12}{13}\right) = 22.6°$

$\tan \angle B = \dfrac{12}{5} = 2.40$ $B = \tan^{-1}\left(\dfrac{12}{5}\right) = 67.4°$

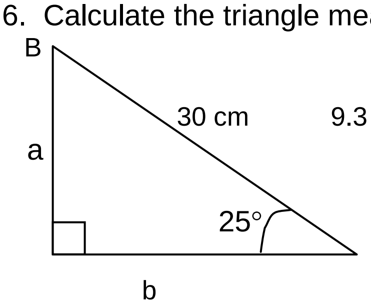
Problems

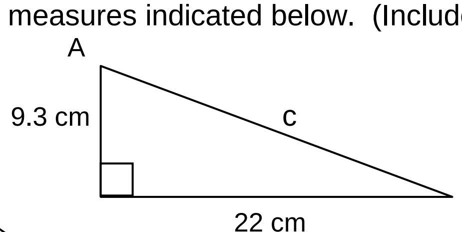






1. For triangle I above, cos A = _______.
2. For triangle I, tan A = _______.
3. For triangle I, A = _______°.
4. For triangle II, c = _______.
5. For triangle III, b = _______.
6. Calculate the triangle measures indicated below. (Include units.)





a = __________ A = __________°

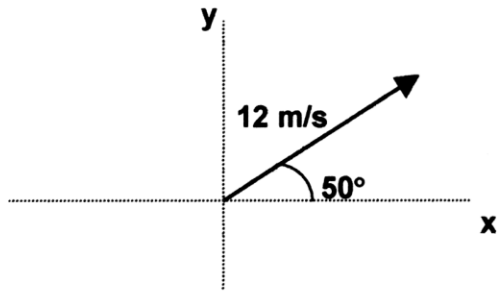
b = __________ B = __________°

c = __________

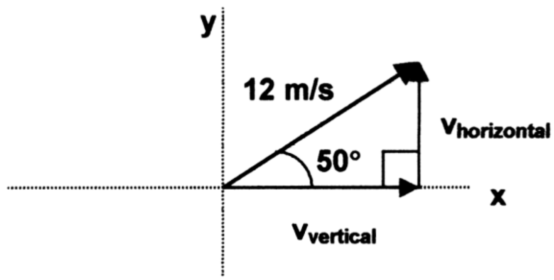
--This is a note page. No questions are asked on this page.--

Vectors

Vectors are quantities that possess both <u>magnitude</u> and <u>direction</u>. Examples of vector quantities include velocity, acceleration, and force. The velocity vector shown below has a speed of 12 m/s and a direction of 50°.



All vectors can be broken up into vertical and horizontal components. These can be found by making a right triangle with the vector as the hypotenuse.



Recall that $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ **and** $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

For the example shown above,

$\sin 50° = \frac{v_{vertical}}{12\ m/s}$

$v_{vertical} = 12\ m/s\ (\sin 50°)$

$v_{vertical} = 9.2\ m/s$

$\cos 50° = \frac{v_{horizontal}}{12\ m/s}$

$v_{horizontal} = 12\ m/s\ (\cos 50°)$

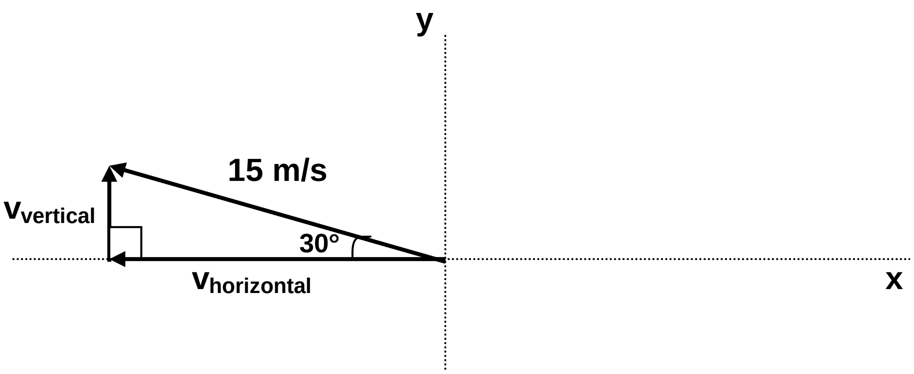
$v_{horizontal} = 7.7\ m/s$

Thus, it can now be seen that an object moving 12 m/s at 50° is simultaneously moving 9.2 m/s vertically and 7.7 m/s horizontally.

Vectors

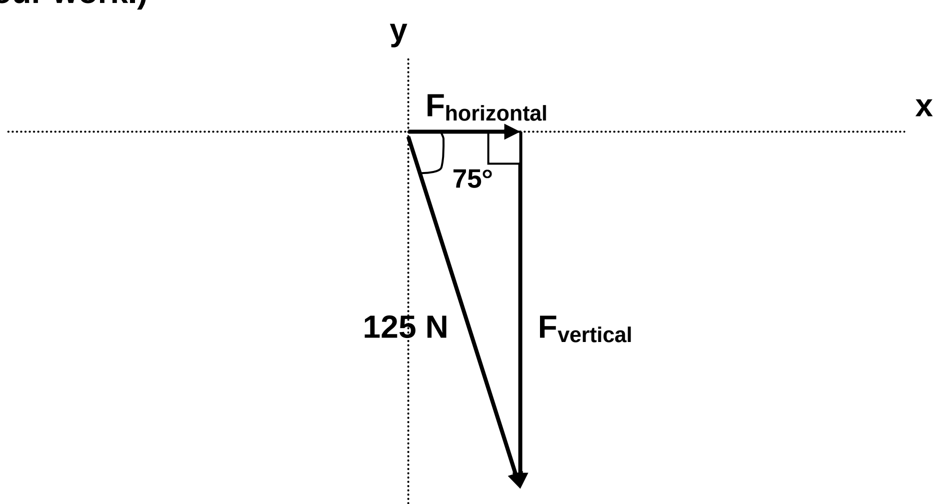
Calculations

1. **Find the horizontal and vertical components of the velocity vector shown below. (Please show your work.)**



a) $v_{vertical}$ = ________________________ m/s **b) $v_{horizontal}$ = ________________________ m/s**

2. **Find the horizontal and vertical components of the *force* vector shown below. (Please show your work.)**



a) $F_{vertical}$ = ________________________ N **b) $F_{horizontal}$ = ________________________ N**